

Mathematics Teachers' Constructions of Circle Theorems in a Dynamic Geometry Environment

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<p>Resumen</p>	<p>Este artículo destaca los detalles de un estudio de desarrollo profesional continuo en el área de la formación del profesorado de matemáticas; con especial atención a las construcciones de los profesores de matemáticas y de construcción de sentido de una variedad de teoremas que involucran arcos de círculo, segmentos, acordes, tangentes y secantes utilizando GeoGebra. Una de las principales conclusiones es que los profesores de matemáticas visualizaron los teoremas de círculo a través de construcciones sobre todo de buen comportamiento naturaleza (apropiadamente limitada), aunque hubo instancias de construcciones prematuros a veces. Mientras que los profesores tuvieron éxito en demostrar y justificar las pruebas visuales de teoremas círculo en el software de geometría dinámica, la confirmación de estas visualizaciones a través de enfoques analíticos demostró ser un reto.</p> <p>Palabras clave: la educación matemática, desarrollo profesional, el software de geometría dinámica, teoremas círculo, visualización</p>
<p>Abstract</p>	<p>This article highlights the details of an ongoing professional development study in the area of mathematics teacher education; with particular attention to mathematics teachers' constructions and sense-making of a variety of circle theorems involving arcs, segments, chords, tangents, and secants using GeoGebra dynamic geometry software. One of the main findings is that mathematics teachers visualized the circle theorems via constructions mostly of well-behaved (appropriately constrained) nature, although there were instantiations of premature constructions at times. Whereas teachers were successful in demonstrating and justifying the visual proofs of circle theorems on the dynamic geometry software, confirmation of these visualizations via analytical approaches proved challenging.</p> <p>Keywords: mathematics teacher education, professional development, dynamic geometry software, circle theorems, visualization</p>
<p>Resumo</p>	<p>Este artigo destaca os detalhes de um estudo de desenvolvimento profissional contínuo na área da formação de professores de matemática; com especial atenção para construções de professores de Matemática e de tomada de sentido de uma variedade de teoremas envolvendo arcos de círculo, segmentos, acordes, tangentes e secantes, utilizando GeoGebra. Uma das principais conclusões é que os professores de matemática visualizaram os teoremas sobre círculo através de construções principalmente de (apropriadamente restrita) natureza bem-comportado, embora às vezes houvesse instâncias de construções prematuras. Enquanto que os professores foram bem</p>

sucedidos em demonstrar e justificar as provas visuais de teoremas sobre círculo no software de geometria dinâmica, a confirmação destas visualizações através de abordagens analíticas provou ser desafiador.

Palavras-chave: educação matemática de professores, desenvolvimento profissional, software de geometria dinâmica, teoremas círculo, visualização

1. Appropriately Constrained Constructions in a Dynamic Geometry Software

As sketched on a piece of paper along with its components such as its sides, vertices, and angles; the drawn figure may be taken for granted to be a square perhaps because congruent marks are used to indicate that all four sides are congruent and the right angle symbol is used to denote the right-ness of the angles; or simply because the figure looks like a square. In a dynamic geometry software (DGS), however, even though we think what we draw is a square, our figure may still not be classified as a well-behaved square. “Students want to end up with a construction, not merely a drawing” (Finzer & Bennett, 1995, p.428). In a dynamic geometry environment, a figure falls into one of the four categories: a drawing, an underconstrained, an overconstrained, or an appropriately constrained (well behaved) figure (p.430). A drawing is what most beginner students and teachers would obtain. Fig.1a shows a kite, which, eventually can be shown to be a drawing when one (fig.1b) or more of its vertices are dragged (fig.1c).

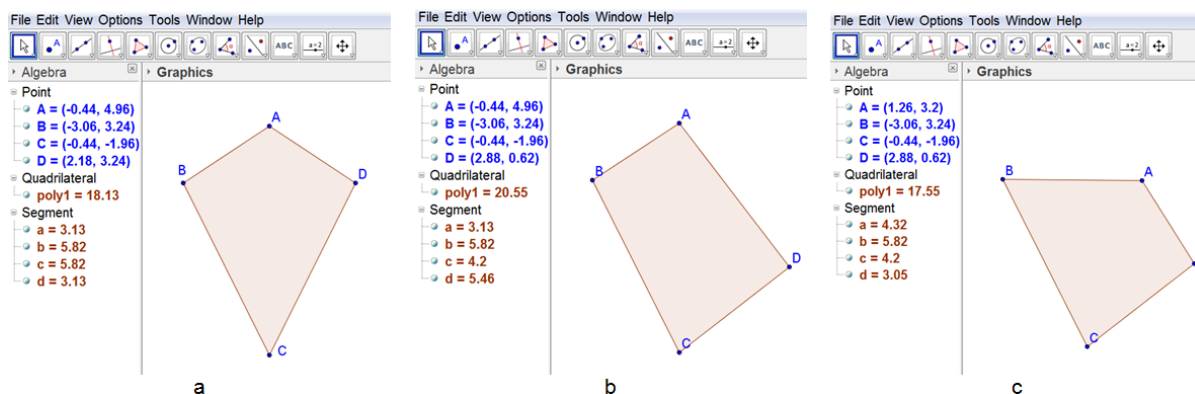


Figura 1. Kite as a Drawing.

An underconstrained figure is a geometric object that possesses one or more of the properties of the geometric figure that is intended to be constructed. Such a figure can be shown to be incorrect by dragging one or more of its vertices. Fig.2a shows an underconstrained kite produced via the two perpendicular lines on which its intersecting diagonals are contained. When Point C is dragged up and down, the display may give the impression that it is always a kite (fig.2b), however, when some other points are dragged; yet the figure ceases to be a kite (fig.2c), it still possesses some of the properties of a kite (e.g., it is still a quadrilateral with perpendicular diagonals).

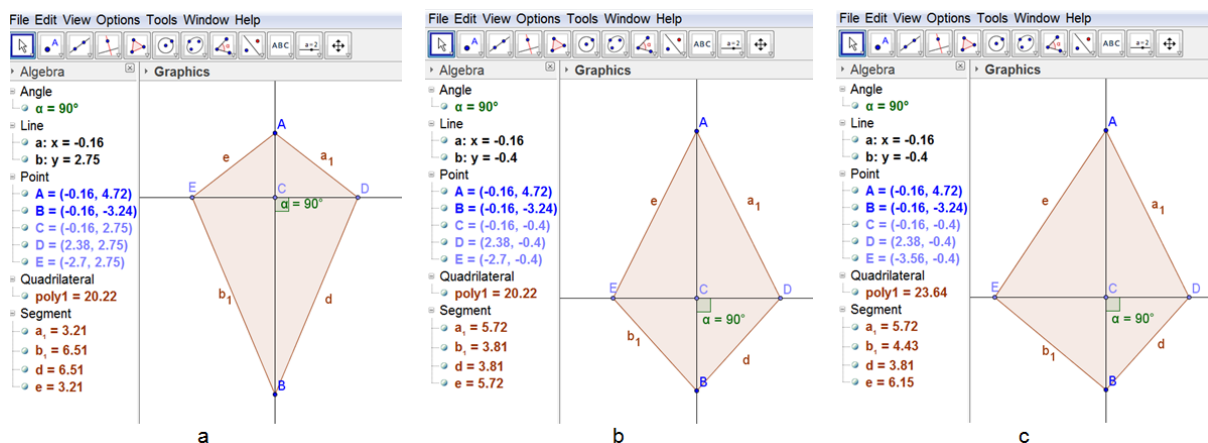


Figure 2. An underconstrained kite.

In an overconstrained figure, the constructor is so picky about imposing excessive constraints on the figure that the figure cannot be explored at all: however the vertices are dragged, the components of the figure (e.g., sides, angles) do not vary and the figure becomes unmanageable. Fig.3a shows an overconstrained kite that is produced as follows. Line segment AB with fixed length 6 units is drawn. Circles centered at A and B with fixed respective lengths 3 and 5 are drawn. The intersection points C and D of the circles are formed. Finally kite ACBD is formed (fig.3a). The figure, when one or more of its vertices are dragged, remains not only a kite, but also a kite that is always congruent to the original one (fig.3b).

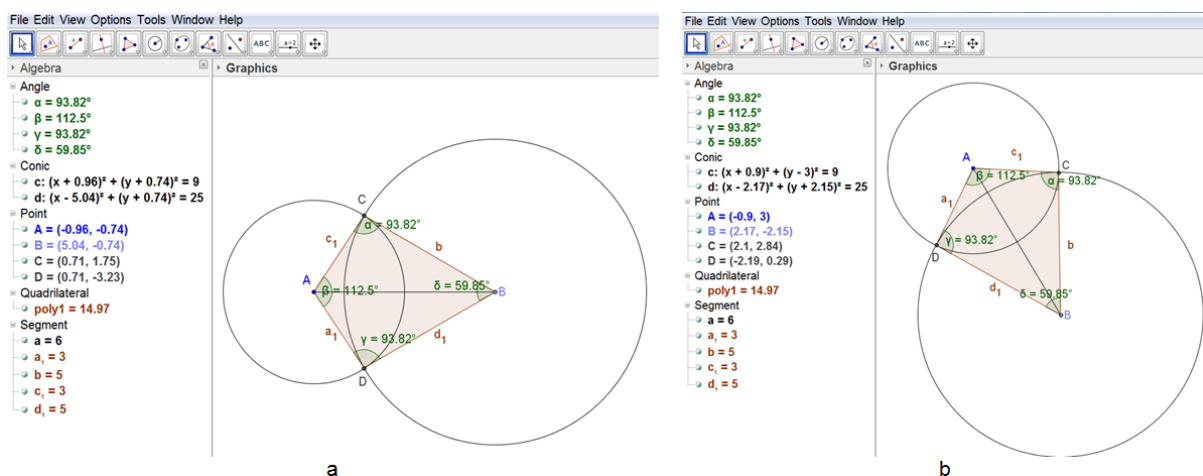


Figure 3. An overconstrained kite.

An appropriately constrained (well-behaved) kite could be formed by intersecting the radius and a chord of a circle that is perpendicular to the radius of the circle (fig.4a). It is worth noting that if the diameter is used instead of the radius, one would still obtain a kite, but a particular one, namely one in which the congruent pair of opposite interior angles are right angles (fig.4b).

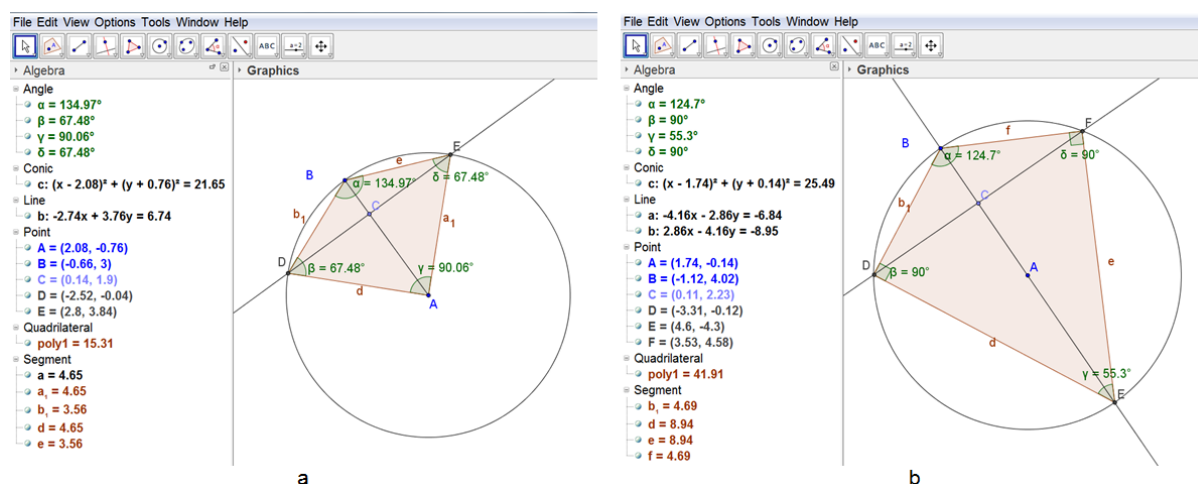


Figure 4. A well-behaved kite.

2. Context and Methodology

Qualitative interview data (Patton, 2002) were collected over two years in a U.S. university as part of a professional development research project titled *Mathematics Teachers' Geometry Connections via Physical Manipulatives, Interactive Applets, and Dynamic Geometry Software*. The connections project was designed to enhance in-service mathematics teachers' content knowledge of mathematics, with particular focus on geometry and connections with geometry using technology (properties of polygons—including triangles, quadrilaterals, and regular polygons; discovering traditional and alternative area formulas; circle theorems; congruence and size transformations; geometric representations of complex numbers; fraction operations with pattern blocks). All professional development sessions were videotaped using one camera, with primary focus on teachers' constructions and interactions with physical and virtual manipulatives along with gestures and inscriptions.

2.1. Rationale for the Study

The categorization suggested by Finzer and Bennett (1995) seems to apply not solely to quadrilaterals, but to various mathematical objects that can be constructed and visualized on a dynamic geometry environment, as this present study attempts to demonstrate. Circle theorems are at the core of the domain of geometry, which constitutes an essential field in the high school mathematics curriculum (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000). Geometric objects such as plane figures seem to be categorized as drawings, underconstrained, overconstrained, or appropriately constrained figures in a dynamic geometry environment; however, we do not know about the significance of geometric objects when they are utilized to investigate and explore circle theorems and connections, which will be the main contribution of the present report. The present study also suggests a framework on mathematics teachers' knowledge of an important domain of geometry, with particular attention to the geometric objects

helping understand various properties of arcs, segments, chords, radii, tangents, and secants.

2.2. Participants and Procedure

Doerr and Pratt (2008) distinguished between exploratory models (those made by an expert) and expressive models (those made by the learner). In a similar line of thinking, Zbiek, Heid, Blume and Dick (2007) postulated that “when students are given a procedure to carry out, they are engaging in exploratory activity; however, when students decide which procedures to use they are engaging in expressive activity” [6]. The data for the analysis of circle theorems, which is the focus of the present report, came from the videotapes of four sets of two-hour interview sessions in a computer lab that included five secondary mathematics teacher volunteers, who worked on the expressive activities in two groups. In the present report, Group A denotes the group of teachers T1, T2, T3 who shared a computer; Group B denotes teachers T4, T5 who shared a computer. The participants were only provided with the statement of each circle theorem from the geometry textbook by Michael Serra (2008); it was up to them to decide which procedures to follow, making all activities to be of expressive type. The qualitative interviews were based on a semi-structured interview model (Bernard, 1994); participants demonstrated their solutions for each problem with the GeoGebra dynamic software. The interview tasks were designed taking into account the construction of circle theorems involving (i) central angles and inscribed angles, (ii) chords and perpendicular lines, (iii) secants and tangents, (iv) triangle centers and cyclic quadrilaterals. All interviews were transcribed; the analyses of the interviews are based on thematic analysis (Boyatzis, 1998) and constant comparison methodology (Glaser & Strauss, 1967).

3. Results and Analysis

The results are presented in four main categories: Theorems involving (i) central and inscribed angles; (ii) chords and perpendicular lines; (iii) tangents and secants; (iv) triangle centers and cyclic quadrilaterals.

3.1. Theorems Involving Central and Inscribed Angles

3.1.1. Inscribed Angle Theorem

The illustration of the Inscribed Angle Theorem [**The measure of an angle inscribed in a circle is one-half the measure of the central angle intercepting the same arc**] involved multiple parts. In the first part, both groups of teachers came up with appropriately constrained (well-behaved) constructions illustrating the inscribed and the central angles intercepting the same arc. All teachers measured the inscribed and the central angles, however, Group A teachers were contented that the measure of the central angle seemed to appear twice the measure of the inscribed angle, without further calculations. Group B teachers used the input bar to actually calculate the ratio of the measure of the central angle to the measure of the

inscribed angle (by typing β / α , which appeared as $f = 2$ in the Algebra View), as illustrated in fig.5a. Upon performing the drag test, T4 emphasized the multiple ways that the inscribed angle theorem can be visualized (fig.5b) while stating “Oh that is what I am used to seeing a lot in textbooks” (fig.5c). T5 remarked that the drag test was successful for the geometric construction **in coordination with** the ratio that was constant at all times.

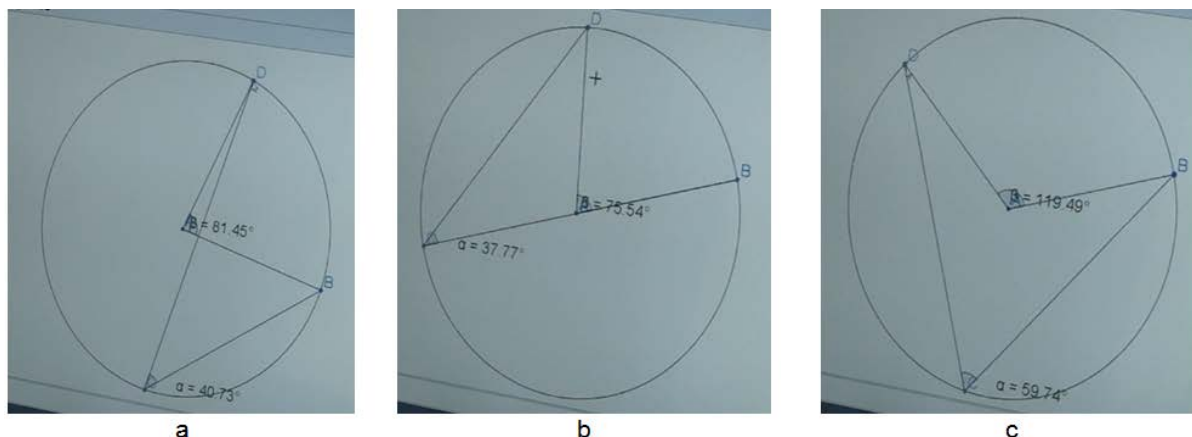


Figure 5. Visualizing the inscribed angle theorem.

3.1.2. Inscribed Angles Intercepting Congruent Arcs Theorem

Inscribed Angles Intercepting Congruent Arcs Theorem [**Inscribed angles that intercept the same arc are congruent**] was also a straightforward one to visualize. What surprised the author of this report was the fact that both groups of teachers constructed two inscribed angles that intercept the same arc (instead of just constructing one inscribed angle and using the software’s drag feature). Group A teachers came up with the construction illustrated in fig.6a and used the drag test with both inscribed angles to show the well-behaved character of their construction. Upon the interviewer’s probing why they were dragging the vertex of the inscribed angle, T1 responded “We’re testing to see if it’s actually true that any angle at the circumference are equal, so not just those specific angles, but any angle at the circumference will be equal.” T3 contributed to the illustration by suggesting drawing the intercepted arc in a different color (using the *Circular Arc with Center between Two Points* Tool) that was shared by the two inscribed angles (fig.6b). T2 noted the theorem’s connection with the Inscribed Angle Theorem [**The measure of an angle inscribed in a circle is one-half the measure of the central angle**] and suggested measuring the central angle as well. Yet they did not construct the intercepted arc (fig.6c), Group B teachers still recognized the theorem’s connection with the Inscribed Angle Theorem by typing β / α , which appeared as $f = 2$ in the Algebra Sidebar.

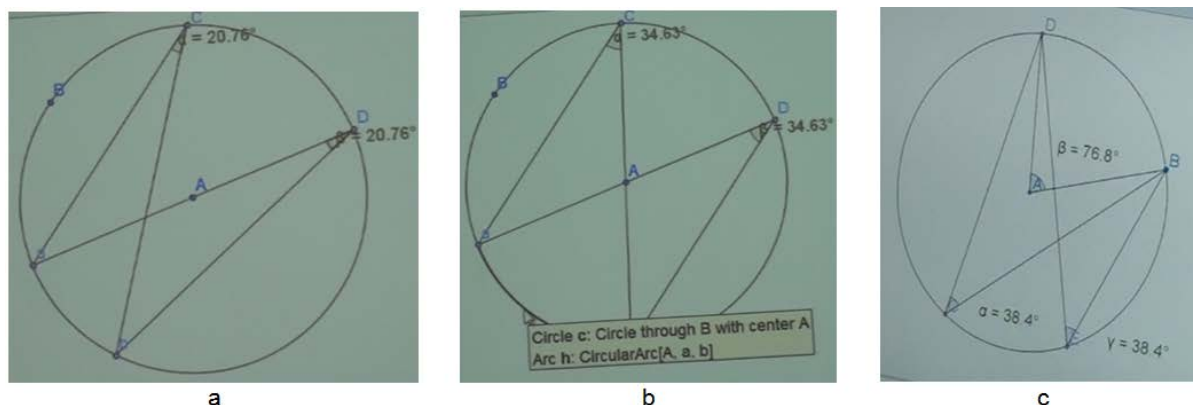


Figura 6. Visualizing the inscribed angles intercepting congruent arcs theorem.

3.2. Theorems Involving Chords and Perpendicular Lines

3.2.1. Thales Theorem

The visualization of the Thales Theorem [**Angles inscribed in a semicircle are right angles**] was one of the easiest constructions for all participating teachers. All teachers came up with well-behaved constructions, however, the procedures they followed differed. Upon suggestion by T1, Group A teachers started with a circle, followed by the construction of a line passing through the center of the circle. They then constructed two chords forming the inscribed angle; measured the angle between the chords as 90° as illustrated in fig.7a. When T2 dragged the vertex of the inscribed angle to the other semicircle, the angle measurement indicated 270° (fig.7b), which did not surprise the teachers as they agreed that **the angle of interest** still measured 90° . Group B teachers started their construction in a totally different way, by building on their construction from the Inscribed Angles Intercepting Arcs theorem. T5 emphasized that Thales Theorem would stand for a particular case of the Inscribed Angles Intercepting Arcs theorem. In agreement with T5, T4 commented “if the central angle is a straight angle then that means that the two inscribed angles have to be 90 degrees” (fig.7c).

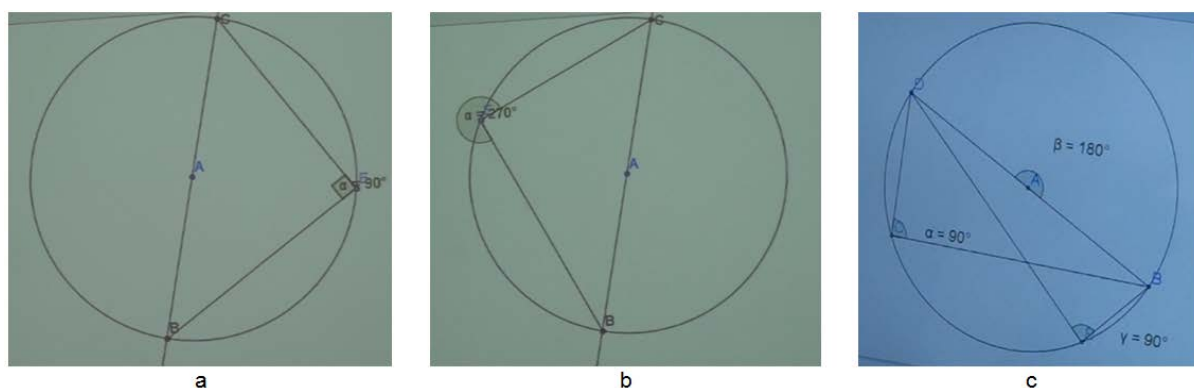


Figura 7. Visualizing Thales theorem.

3.2.2. Perpendicular Bisector of a Chord Theorem

In the visualization of Perpendicular Bisector of a Chord Theorem [**The perpendicular bisector of a chord passes through the center of the circle**], Group A teachers started their construction with a circle, followed by an arbitrary chord on the circle using the *Line Segment Tool*. They completed their construction via the built-in *Perpendicular Bisector Tool*; they remarked that the line would always contain the center of the circle wherever the chord is dragged (fig.8a). T1 further commented: “if we did this construction on the paper using a compass, two congruent arcs centered at the endpoints of the chord would intersect at two points... above and below the chord... and when we draw the line it would go through the center of the circle.” Group B teachers’ visualization was rich in ideas in the sense that it illustrated more than just the Perpendicular Bisector of a Chord Theorem. They started by constructing two congruent chords (two chords that appeared congruent – these were in fact underconstrained figures); followed by the construction of two perpendicular bisectors. Both teachers indicated that it was an obvious observation that the perpendicular bisectors would contain the center of the circle. They further remarked that the chords would (i) trace out congruent central angles [**Chord Central Angle Theorem: If two chords in a circle are congruent, then they determine two central angles that are congruent**]; (ii) intercept congruent arcs [**Chord Arcs Theorem: If two chords in a circle are congruent, then they determine two central angles that are congruent**]; (iii) be equidistant from the center of the circle [**Chord Distance to Center Theorem: Two congruent chords in a circle are equidistant from the center of the circle**] (fig.8b).

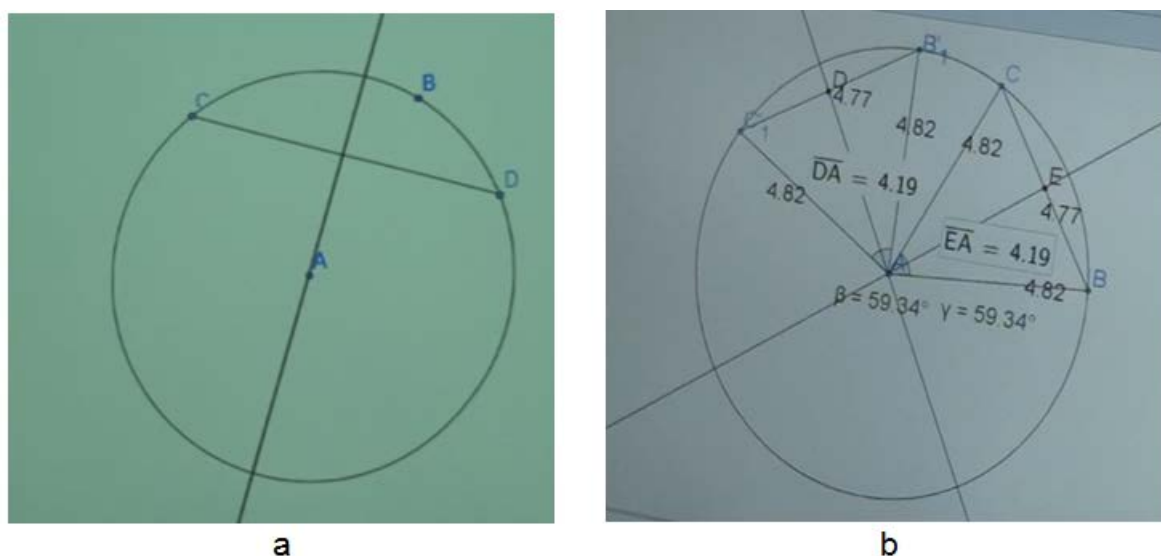


Figura 8. Visualizing the Perpendicular Bisector of a Chord Theorem.

3.3. Theorems Involving Tangents and Secants

3.3.1. Tangents Theorem

Tangents Theorem [**A tangent to a circle is perpendicular to the radius drawn to the point of tangency**] probably was the easiest construction among all circle theorems for all the teachers. This theorem stood as rich in context in terms of

triggering Group A teachers' reflective reasoning. After constructing a circle and one radius, Group A teachers visualized this theorem by constructing the tangent line using GeoGebra's built-in *Tangents Tool* (fig.9a). T2 and T3 remarked that this theorem could somehow be related with Thales Theorem they previously explored. In an attempt to justify their thinking, teachers decided to work it out analytically. T1 and T2 came up with drawings relating the two theorems, however, none of them could do the finishing touch (fig.9b-c). Upon T1's drawing (fig.9b), T3 commented that line segments AD, AB, and AC would be congruent and suggested labeling the congruent angles of the isosceles triangles and that would lead to the proof of the theorem.

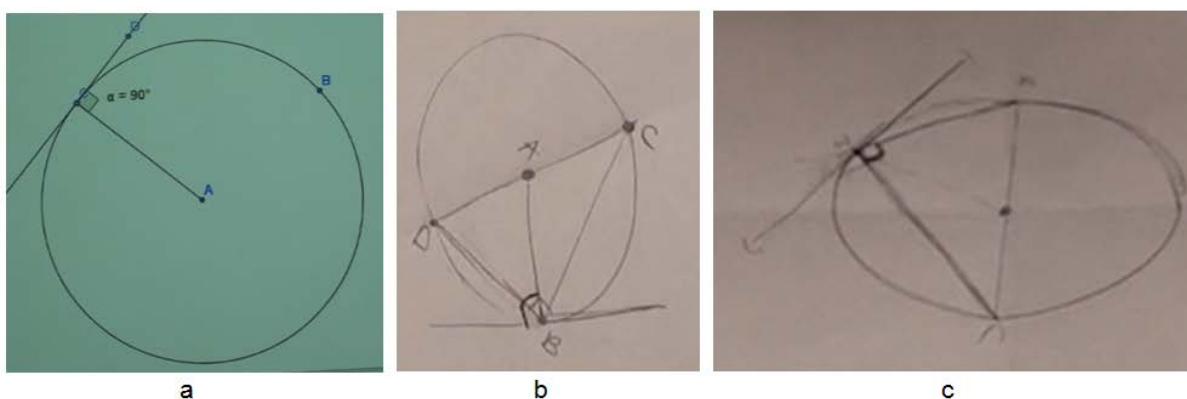


Figura 9. Group A Teachers' Visualization of Tangents Theorem.

3.3.2. Tangent Segments Theorem

Tangent Segments Theorem [**Tangent segments from a point to a circle are congruent**] was another easy construction that followed the Tangents Theorem. Both groups of teachers followed totally different approaches in their visualizations. Group A teachers started by constructing the two tangent lines drawn to two arbitrary points on the circle. Upon forming the intersection of the tangent lines, T2 suggested constructing the line segments connecting the point of intersection to the points of tangency. T1 remarked that the tangent segments would be equal, once their numerical values d and e appeared on the Algebra Sidebar (fig.10a). Upon the interviewer's probing why the tangent segments are equal, T2 remarked that by drawing the two radii she would obtain a kite; she suggested drawing the diagonals and measuring the angle between the diagonals while she commented "Now just two diagonals CD and AE they should be... if it's a kite, they will be perpendicular" (fig. 10b). T3 remarked that the non-congruent angles of the kite would be supplementary, without realizing that this was actually another theorem about tangent segments, that is, the angles version of the Intersecting Tangents Theorem [**The angle formed by two tangents and the central angle corresponding to the smaller intercepted arc are supplementary angles**]. Group B teachers used the Intersecting Secants Theorem to deduce the Intersecting Tangents theorem, as described in the following section.

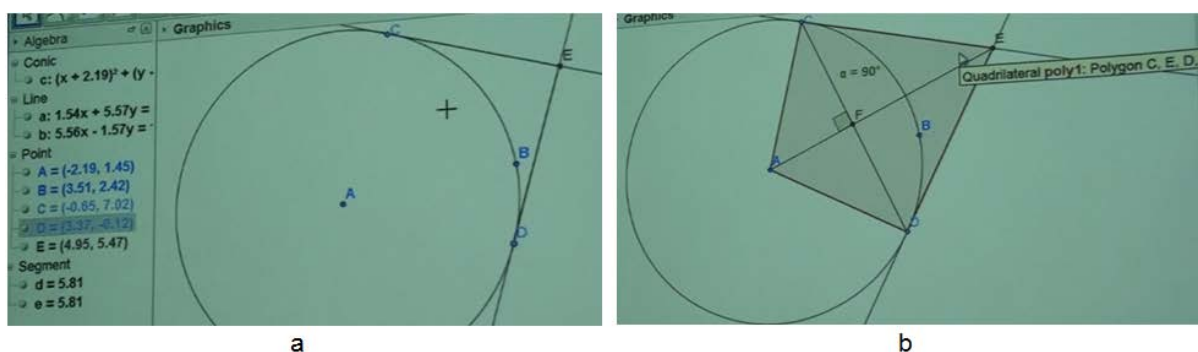


Figure 10. Visualizing Tangent Segments Theorem.

3.3.3. Intersecting Secants (Power of Exterior Point) Theorem

Power of Exterior Point Theorem applies when two secants intersect outside a circle. Group B teachers not only came up with a well-behaved construction illustrating this theorem (fig.11a), but they also used this theorem to deduce the Intersecting Tangents Theorem via a limiting strategy. In fig.11a, the variables d and e in the Algebra View stand for the products CE by CD , and CF by CB , respectively. When T4 was dragging Point C to test whether d and e would equal all the time, she was surprised to see that the equality would hold even when Point C was in the interior of the circle (fig.11b). What T4 discovered was actually the Intersecting Chords (Power of Interior Point) Theorem [If two chords intersect in a circle, then the product of the segment lengths on one chord is equal to the product of the segment lengths on the other chord]. T5 later obtained the Intersecting Tangents Theorem by applying a limiting strategy that was not used by any other teacher: He dragged Point E in such a way that Points E and D at the top coincided (fig.11c), thus verifying the Intersecting Secant-Tangent Theorem. Finally, he dragged Point F such that Points F and B at the bottom coincided, respectively.

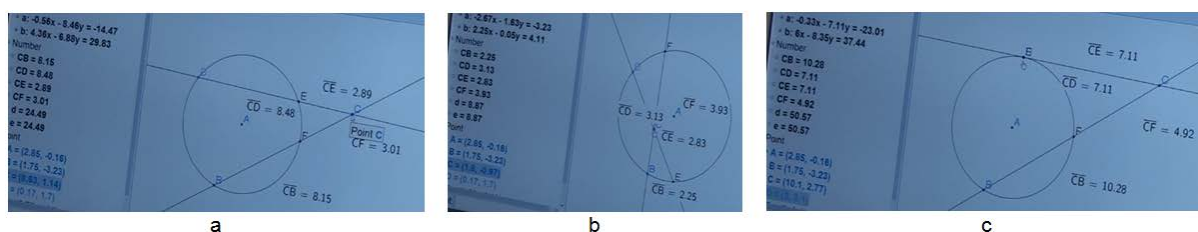


Figure 11. Group B Teachers' Visualization of Intersecting Secants Theorem.

3.4. Theorems Involving Triangle Centers and Cyclic Quadrilaterals

3.4.1. Cyclic Quadrilateral Theorem

Cyclic Quadrilateral Theorem [The opposite angles of a cyclic quadrilateral are supplementary] stood as an important construction for the Group A teachers in the sense that for the first time, they made use of the input bar to actually verify the statement of a circle theorem. They started with a circle and obtained their

quadrilateral by placing four arbitrary points on the circle and connecting them via line segments. T2 then measured the angles; T1 suggested using the input bar by actually typing the sum of the opposite angles (She typed $\alpha + \beta$, which appeared as $\epsilon = 178.82^\circ$ in the Algebra View). This approach helped the teachers realize that their cyclic quadrilateral construction was incorrect (fig.12a); T1 commented that Point D was not on the circle by dragging it in and out of the circle (fig.12b). Upon fixing the cyclic quadrilateral construction, teachers redid the calculations and verified that their construction was correct. While performing the drag test, T2 remarked that the opposite angles were always supplementary. T1 further observed: "So there're two angles that are not changing, while the other two are changing. So, the two that are changing are always changing such that they add up to 180." This activity seemed to trigger a lot of enthusiasm in both groups of teachers.

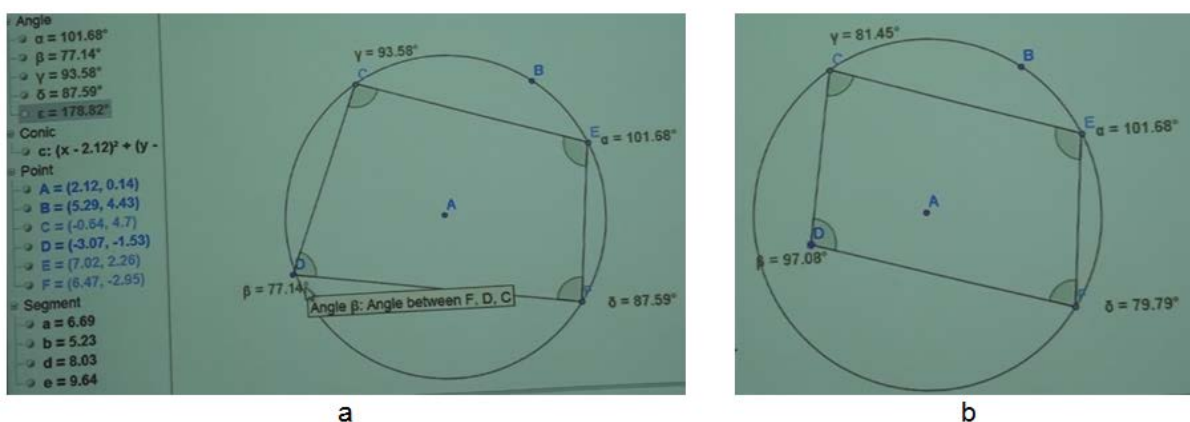


Figure 12. Group A Teachers' Visualization of Cyclic Quadrilateral Theorem.

Although Group B teachers did not use the input bar to verify the Cyclic Quadrilateral Theorem, their visualization seemed to be at a more advanced level than Group A teachers' construction for two main reasons. First, Group B teachers thought about this theorem in connection with the previous theorems involving inscribed and central angles. Both teachers recognized this connection, and they made use of it by actually constructing not only the required cyclic quadrilateral, but also two of the relevant central angles as well (fig.13a). Second, this activity triggered teachers' reflective reasoning in a manner that led T4 to want to analytically prove this theorem. Among all circle theorem explorations, this is the only occurrence in which a teacher algebraically proved a circle theorem (fig.13b).

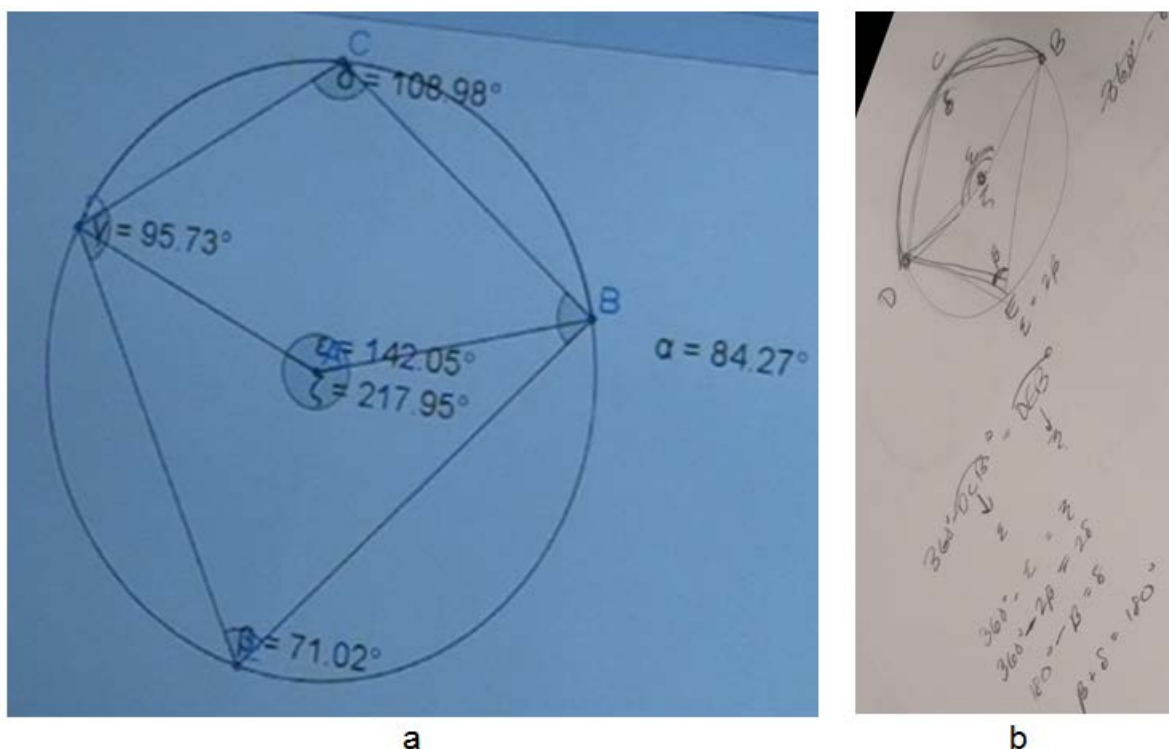


Figura 13. Group B Teachers' Visualization of Cyclic Quadrilateral Theorem.

3.4.2. Circle Theorems Involving Inradius, Circumradius, Semi-Perimeter

Formula based theorems stood as challenging for both groups of teachers as they were required to pay attention to well-behaved character of their constructions along with the formulas appearing on the Algebra Sidebar. Visualizing triangle area formula $A = s \cdot r$ (here s denotes the semi-perimeter of the triangle and r denotes the radius of the inscribed circle) was the most challenging one in that teachers had hard time figuring out how to construct the inradius r of the triangle; it was necessary to see its connection with the Tangents Theorem they previously explored (fig.14a). Among all formulas, $A = abc/4R$ (here a, b, c denote the lengths of sides of the triangle and R denotes the radius of the circumscribed circle) offered a variety of visualizations as teachers came up with the circumscribed circle construction in multiple ways: (i) via the traditional intersecting the perpendicular bisectors approach; (ii) via the built-in *Circle through Three Points Tool*. Group A teachers emphasized the importance of using the parentheses syntax appropriately when typing the expression $abc/4R$ in the input bar (fig.14b).

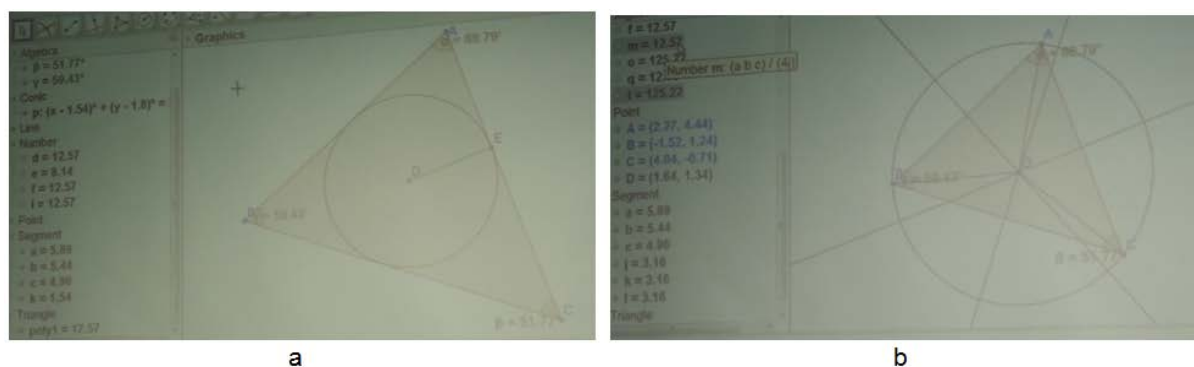


Figura 14. Group A Teachers' Visualization of Triangle Area Formulas.

3.4.3. Heron and Brahmagupta's Formulas

Prior to the constructions of the formula based circle theorems on the software, Group B teachers often made sketches on the paper (fig.15a). As was the case with Group A teachers, Group B teachers initially thought that the inscribed circle would be tangent to the triangle through the intersection of the three angle bisectors with the three sides (fig.15a). Teachers in both groups were able to test and falsify this conjecture using the DGS. All teachers realized how Heron's formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ (here a, b, c denote the lengths of sides of the triangle and s denotes the semi-perimeter) and Brahmagupta's formula $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$ (here a, b, c, d denote the lengths of sides of the cyclic quadrilateral and s denotes the semi-perimeter) were connected by using a limiting process. Fig.15b illustrates Group B teachers' illustration of Brahmagupta's formula. Upon the interviewer's probing how they would deduce Heron's formula, T4 indicated that Heron's formula would be a special case of Brahmagupta's formula by stating "you would essentially have to take off a side like this... There it goes. You are going from a quadrilateral to a triangle and I just shifted it over. So now the length of f is zero" (fig.15c). T5 further tested Heron's formula for the "triangle" they obtained in fig.15c by entering the formula in the input bar and obtained the same area value 17.21 that appeared in the Algebra View.

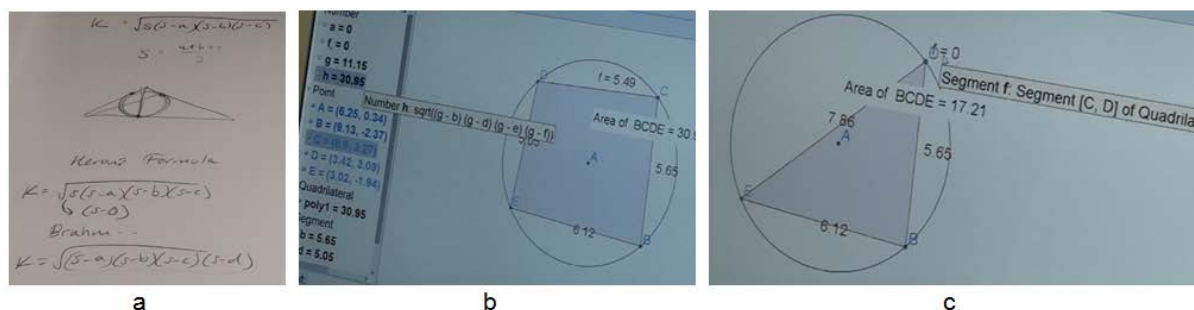


Figura 15. Group B Teachers' Visualization of Heron and Brahmagupta's Formulas.

4. Discussion and Conclusion

Through expressive activities, the mathematics teacher participants came up with multiple ways that circle theorems could be visualized in a dynamic geometry environment. DGS certainly stood as a helpful asset in the professional development of the participating teachers. At times, some circle theorems were so obvious that teachers sufficed with verbal justification. Although they were not asked – nor required – to do so, they occasionally felt the need to provide analytic justification in coordination with the theorems they represented with the DGS. As presented in this report, visualization of the circle theorems via expressive activities seemed to offer multiple ways by which the DGS triggered teachers' reflective and intuitive reasoning along with a desire for further discoveries and connections among circle theorems within the geometry strand. Framed in the view of experimental mathematics (Borwein & Bailey, 2003; Borwein, 2005; Sinclair, 2008), all five teacher participants can be considered to use the DGS primarily for verifying the circle theorems, "gaining insight and intuition, discovering new patterns and relationships, and graphing to expose math principles" (Borwein, 2005, p.76). In the formula-based circle theorems, and when a circle theorem could be deduced using another circle theorem, teachers seemed to use the DGS for "testing and falsifying conjectures, and discovering new patterns and relationships" (p.76); a few rich-in-context instantiations illustrated teachers' utilization of the DGS for "exploring a possible result to see if it merits formal proof, and for suggesting approaches for formal proof" (p.76).

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